THE TRANSVERSE MOTION OF SOLID PARTICLES IN A FLOW WITH FLUCTUATING SHEAR

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We calculate the specific lift which must be experienced by a spherical particle with density different from that of the fluid, in a flow with fluctuating shear.

A solid particle in a shear flow (i.e., when the longitudinal velocity has a transverse gradient) moves with the fluid and at the same time rotates; in turn its rotation gives rise to a definite perturbation of the flow about the particle. The interaction between the rotation of the particle and the fluid surrounding it, and the fundamental flow can give rise, as many observations [1-3] show, to a transverse motion of the particle so that its trajectory deviates from the streamline of the unperturbed flow.

This effect depends on a whole series of factors – the shape and deformability of the particle, the velocity profile of the unperturbed flow, whether or not the flow is steady, the wall effect, etc. The number of different combinations of these factors is extremely great; hence in the literature only a limited number of different cases is discussed or studied experimentally.

Thus, the effect of the transverse displacement of rigid spherical particles in the flow of a non-Newtonian fluid was discussed in [4], while that of deformable particles in a Newtonian fluid was considered in [5].

Even if we restrict ourselves to the consideration of only rigid spherical particles in a Newtonian fluid, there are many combinations of the different factors, each of which must be considered separately; first we have to distinguish the cases when the resultant force F and moment M on the particle from the fluid are zero or nonzero. Analysis of experimental results shows that we also have to take into account the Reynolds number for the relative motion of the particle, defined as $\mathrm{Re_p} \equiv Va/\nu$, and the relative distance from the wall y/a.

Theoretical calculations were made in [6] of the drag, lift, and moment of the forces on a spherical particle for $\mathrm{Re_p} \ll 1$; this is the case, when $F, M \neq 0$, $y/a \to \infty$. An approximate solution was obtained by representing the velocity and pressure fields as series in powers of $\mathrm{Re_p}$, retaining only terms in $(\mathrm{Re_p})^0$ and $(\mathrm{Re_p})^1$. The solution was constructed separately for the region near the surface of the sphere (in the variables x_i/a) and for the region $x_i/a \gg 1$ (in the variables $X_i \equiv \mathrm{Re_p}(x_i/a)$) with subsequent matching of the asymptotic expressions for both regions. For the lift on a sphere moving with translational velocity $\vec{V}$ and rotating with angular velocity $\vec{\Omega}$ with respect to the fluid an expression of the form

$$F_L = C_L \rho_f \, a^3 [\vec{\Omega} \times \vec{V}], \tag{1}$$

was obtained where $C_L \approx \pi$.

Although (1) was obtained for the rotation of a sphere in an unbounded quiescent fluid and $M \neq 0$, the authors of [6] assume that it can be applied to the case of the rotation of a particle in a shear flow with equilibrium (i.e., when $M = 0$) velocity Ω_p.

An approximate solution was obtained in [7] for the lift on a freely rotating ($M = 0$) sphere in a flow with linear velocity profile (Kutta flow); in this case also an expansion in a series in powers of $(1/\nu)$ was used, retaining terms of zero order and of the first power:

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$$F_L = 81{,}2\,\rho_f\, a^2 (\nu/j)^{1/2} \left[\vec{j} \times \vec{V}\right], \tag{2}$$

where V is the velocity of the sphere with respect to the fluid (on a streamline passing through the center of the sphere); $\vec{j} = \vec{k}\cdot dU_x/dy$ is the shear.

Although the approximate method used in [7] can give rise to objections, simple experiments conform to an equation of the form (1). Thus, in [1] ($M \neq 0$, $F = 0$) it was found that $C_L \approx 18$ for $a = 0.075$ cm, $Re_p = 0.08$ and $y/a > 4$. The rotational velocity of the particle was measured directly and was equal to the theoretical value (for creeping flows)

$$\Omega_p = 1/2\, dU_x/dy.$$

The lift for the case M = 0 was also measured in [8], but the data are for large Re_p and small relative distances from the wall.

A weak transverse displacement of solid spherical particles was also detected for the case F, M = 0 [9]. Evidently this effect depends on the curvature of the velocity profile, d^2U_x/dy^2 [10].

Unsteadiness of the fluid flow can also lead to the appearance of transverse forces on the particle. Below we consider the motion of a solid spherical particle in a fluctuating Kutta flow. Such a flow is an approximate model of the viscous sublayer of a turbulent flow where the shear stress fluctuations reach 30% of the average [11]. In these cases a solid particle with $\rho_p \neq \rho_f$ experiences translational and rotational oscillatory motions shifted in phase with respect to these motions of the fluid; simultaneous translational and rotational motion of the particle with respect to the fluid leads to the appearance of lift.

To calculate this effect (see below) we assume that the Reynolds number for the relative motion is small so that, as shown in [6], the force and moment of the forces on the particle can be calculated from the corresponding equations for creeping flow.

The unperturbed flow of the fluid in the direction of the x-axis is described by the equation

$$U_f = u_f \sin\omega t + j_0 (y - y_0) \sin\omega t. \tag{3}$$

Then the solid particle also oscillates in translation, but shifted in phase

$$U_p = u_p \sin(\omega t + \alpha_1), \tag{4}$$

the amplitude and phase shift α_1 being given by the following equations:

$$\frac{u_p}{u_f} = \left(\frac{1+\beta^2}{1+\beta^2\gamma^2}\right)^{1/2} \approx 1 - \frac{1}{2}\,\frac{\gamma+1}{\gamma-1}\,\mathrm{tg}^2\alpha_1, \tag{5}$$

$$\mathrm{tg}\,\alpha_1 = -\beta\,\frac{\gamma-1}{\beta^2\gamma+1} \approx -\beta(\gamma-1), \tag{6}$$

where

$$\beta \equiv \frac{2}{9}\,\frac{a^2\omega\rho_f}{\mu} \text{ and } \gamma \equiv \frac{\rho_p}{\rho_f},$$

and the approximate equality is valid for a small phase shift. As the spherical particle rotates in the unbounded fluid the moment of the forces on the particle is

$$M = 8\pi\mu a^3 (\Omega_f - \Omega_p). \tag{7}$$

For a shear flow it is assumed that

$$M = -8\pi\mu a^3 (\Omega_p - \Omega_e), \tag{8}$$

where Ω_e is the equilibrium velocity of rotation of the solid particle in a flow with shear j, where $\Omega_e = j/2$ [1]. Then for the rotational oscillations of the particle due to the shear fluctuations $j_0 \sin\omega t$ we can derive amplitude and phase equations similar to (5) and (6):

$$\mathrm{tg}\,\alpha_2 \approx -\frac{1}{15}\,\frac{a^2\omega\rho_p}{\mu} = -\frac{3}{10}\,\beta\gamma_0. \tag{9}$$

Then, by (1), and after time-averaging (for small phase shifts, i.e., $\beta \ll 1$)

$$F_L \approx \frac{7\,C_L\,a^3\rho_f}{40}\,j_0 u_f\,\gamma(\gamma-1)\beta^2, \tag{10}$$

$$\frac{F_L}{p_s} \approx \frac{21}{160}\,\frac{C_L}{\pi}\,\frac{j_0 u_f}{g}\,\gamma\beta^2. \tag{11}$$

Estimates of the values shows that for sufficiently large particles ($a \sim 30$–$40\ \mu$) at high frequency ($\omega \sim 10^3\ \mathrm{sec}^{-1}$) and with $j_0 \sim 10^3\ \mathrm{sec}^{-1}$, $u_f \sim 15$ cm/sec (which corresponds to conditions at the outer boundary of the viscous sublayer when $V_* \sim 6$ cm/sec) the lift may exceed the weight of the particle many times, i.e., give rise to suspension of the dispersed phase.

NOTATION

F, M	are the force and moment of the drag forces;
F_L	is the lift;
V, Ω	are the translational and rotational velocity of the particle with respect to the fluid;
U	is the absolute velocity;
Ω_p, Ω_f	are the absolute angular velocity of the particle and of the fluid;
ρ_p, ρ_f	are the particle and fluid densities;
μ	is the dynamic viscosity;
j	is the shear;
a	is the radius of solid particle;
y	is the distance of the center of the particle from the solid wall;
ω	is the frequency of fluctuations;
V_*	is the dynamic velocity (friction velocity);
Re_p	is the Reynolds number for the relative velocity and radius of the particle;
g	is the acceleration due to gravity.

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